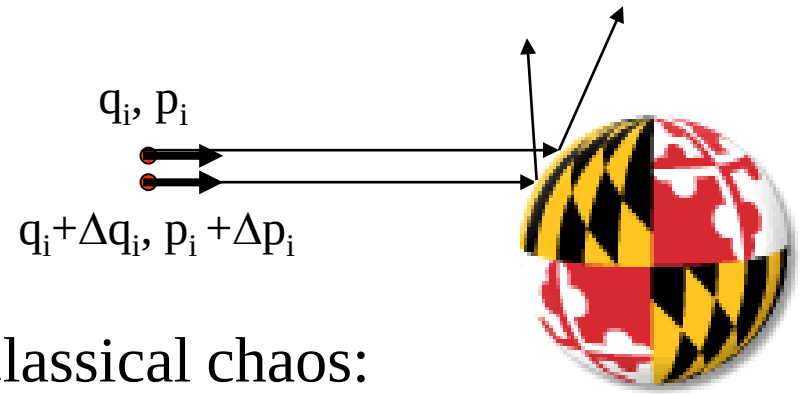
Chaos

Classical: Extreme sensitivity to initial conditions

$$\dot{q}_i = \partial H / \partial p_i$$
$$\dot{p}_i = -\partial H / \partial q_i$$



Manifestations of classical chaos:

Chaotic oscillations, difficulty in making long-term predictions, sensitivity to noise, etc.

Time series, iterated maps, Lyapunov exponents, etc.

Quantum: ???

Heisenberg Uncertainty principle limits knowledge of initial conditions

$\Delta p \, \Delta q > h/2\pi$

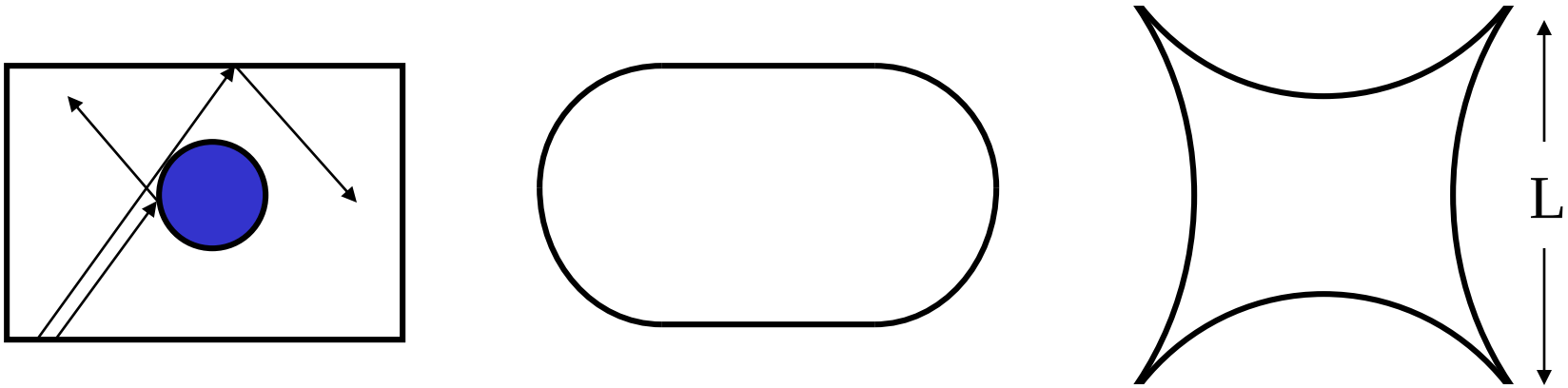
$$\frac{1}{2m}\left(-i\hbar\nabla - qA\right)^2 \Psi + V\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

Manifestations of quantum chaos:

Breaking of degeneracy, Scars, Strong eigenfunction fluctuations

Wave Chaos in Bounded Regions

Consider a two-dimensional infinite square-well potential box that shows chaos in the classical limit:



Now solve the Schrodinger equation in the same potential well
These solutions can be mapped to those of the Helmholtz equation
for electromagnetic fields in a 2D cavity

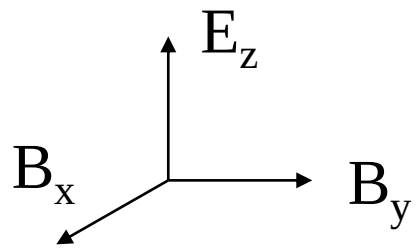
Examine the solutions in the semiclassical regime:
wavelength $\lambda \ll$ system size L

What will happen?

Schrödinger – Helmholtz Analogy

Consider a "two-dimensional" electromagnetic resonator

Only transverse magnetic (TM) modes propagate for f < c/2d ~ 19 GHz, in our case, where d is the height of the cavity



$$\nabla^2 \Psi_n + \frac{2m}{\hbar^2}\left(E_n - V\right)\Psi_n = 0$$

with $\Psi_n = 0$ *at boundaries*

Schrödinger equation

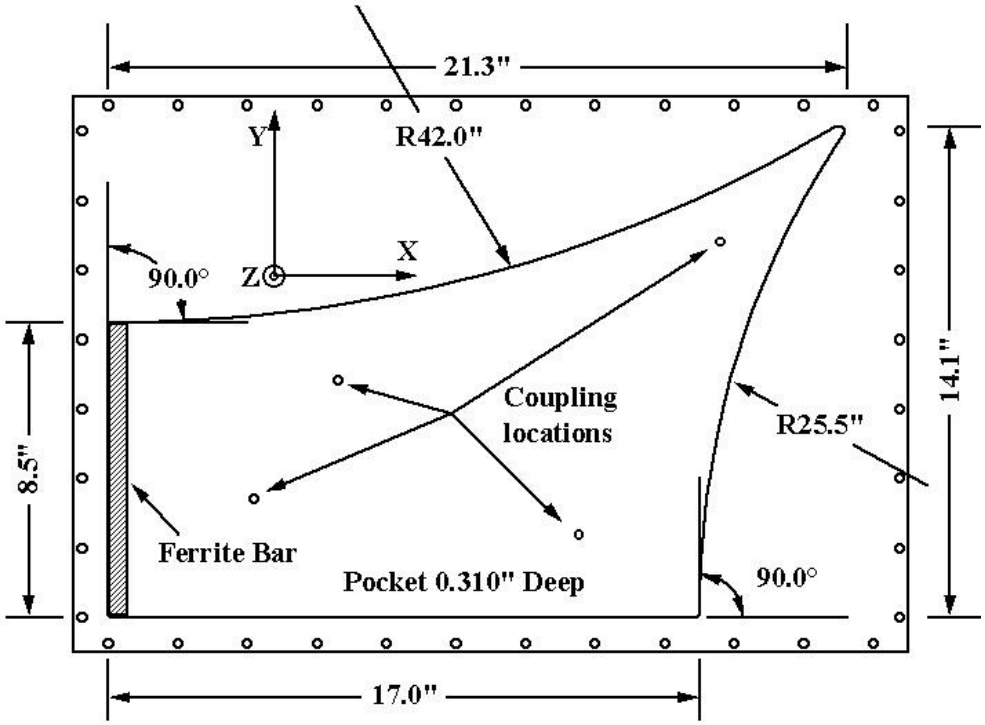
$$\nabla^2 E_{z,n} + k_n^2 E_{z,n} = 0$$

with $E_{z,n} = 0$ *at boundaries*

Helmholtz equation

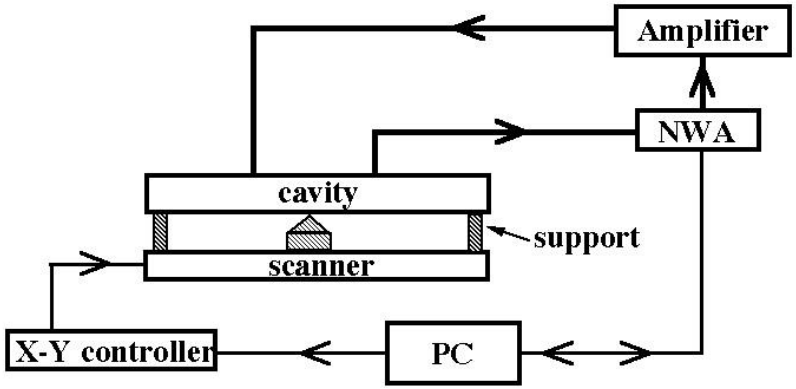
Stöckmann + Stein, 1990

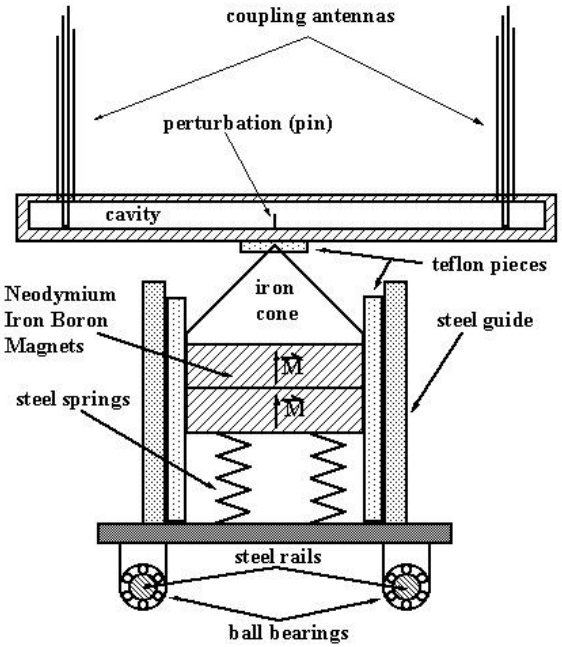
How do we Perform the Experiment?



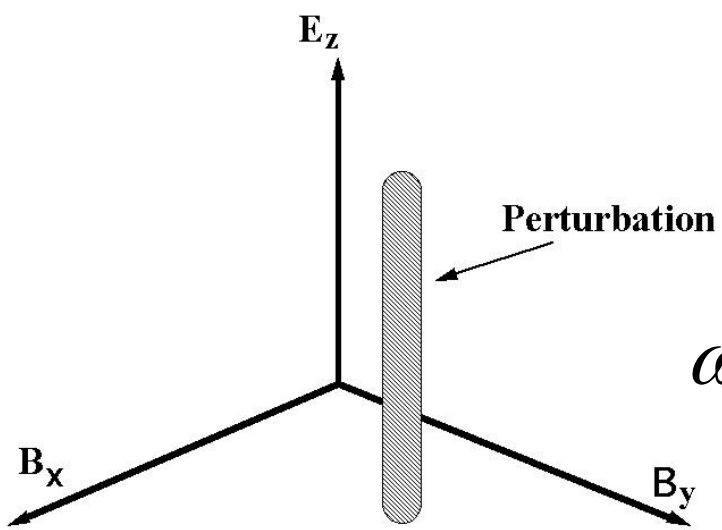
Quarter bow-tie microwave resonator

Measurement setup





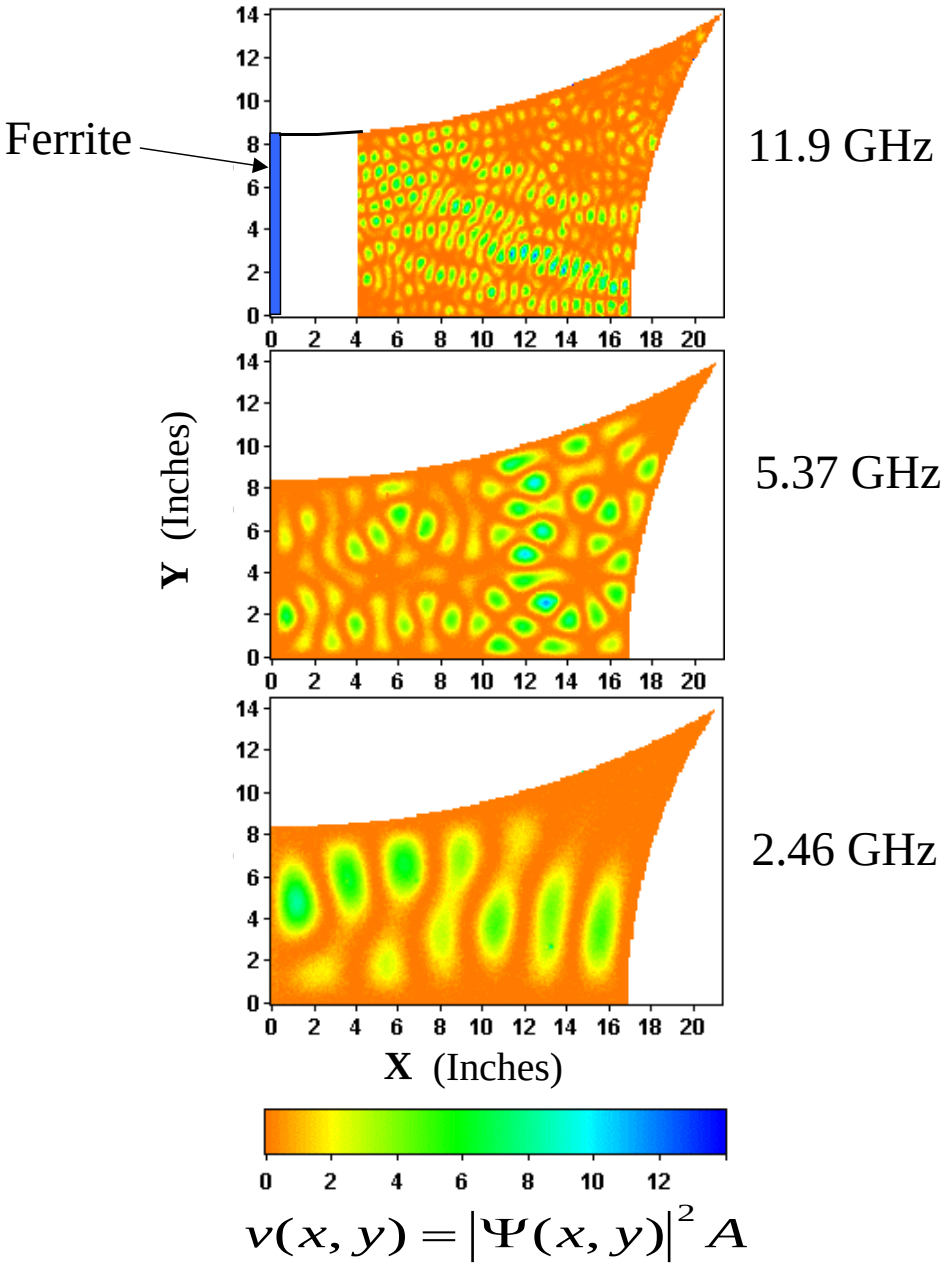
Perturbation scanning system



Measure E_z through cavity perturbation

$$\omega^2 = \omega_0^2 \left(1 + \int \left(|B|^2 - |E_z|^2\right) dV_{pert}\right)$$

Eigenfunctions



$$v(x,y) = |\Psi(x,y)|^2 A$$

A. Gokirmak and S. M. Anlage,
Rev. Sci. Instrum. 69, 3410 (1998).

and

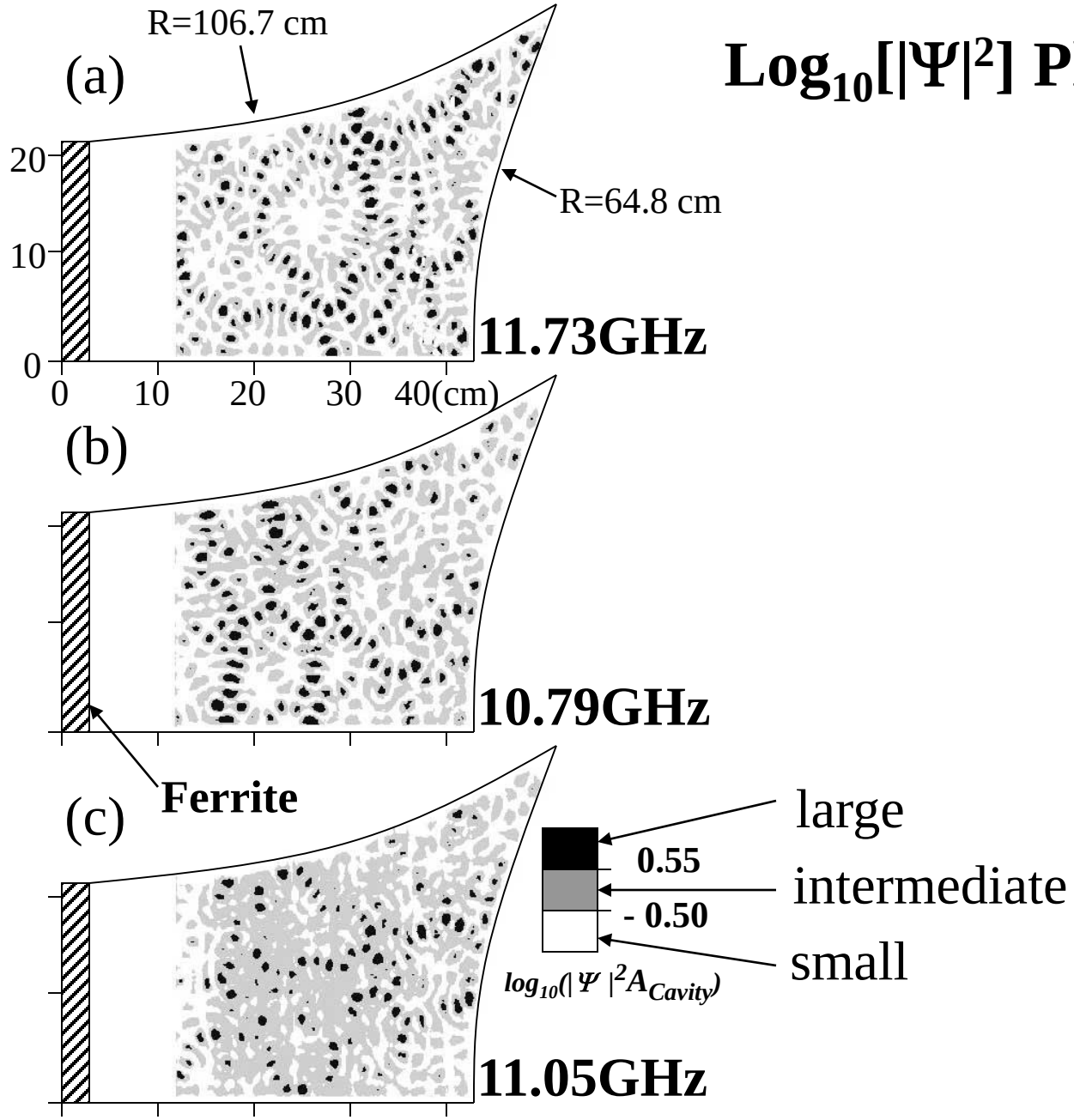
D. H Wu and S. M. Anlage,
Phys. Rev. Lett. 81, 2890 (1998).

$\mathbf{Log_{10}[|\Psi|^2]}$ Plots

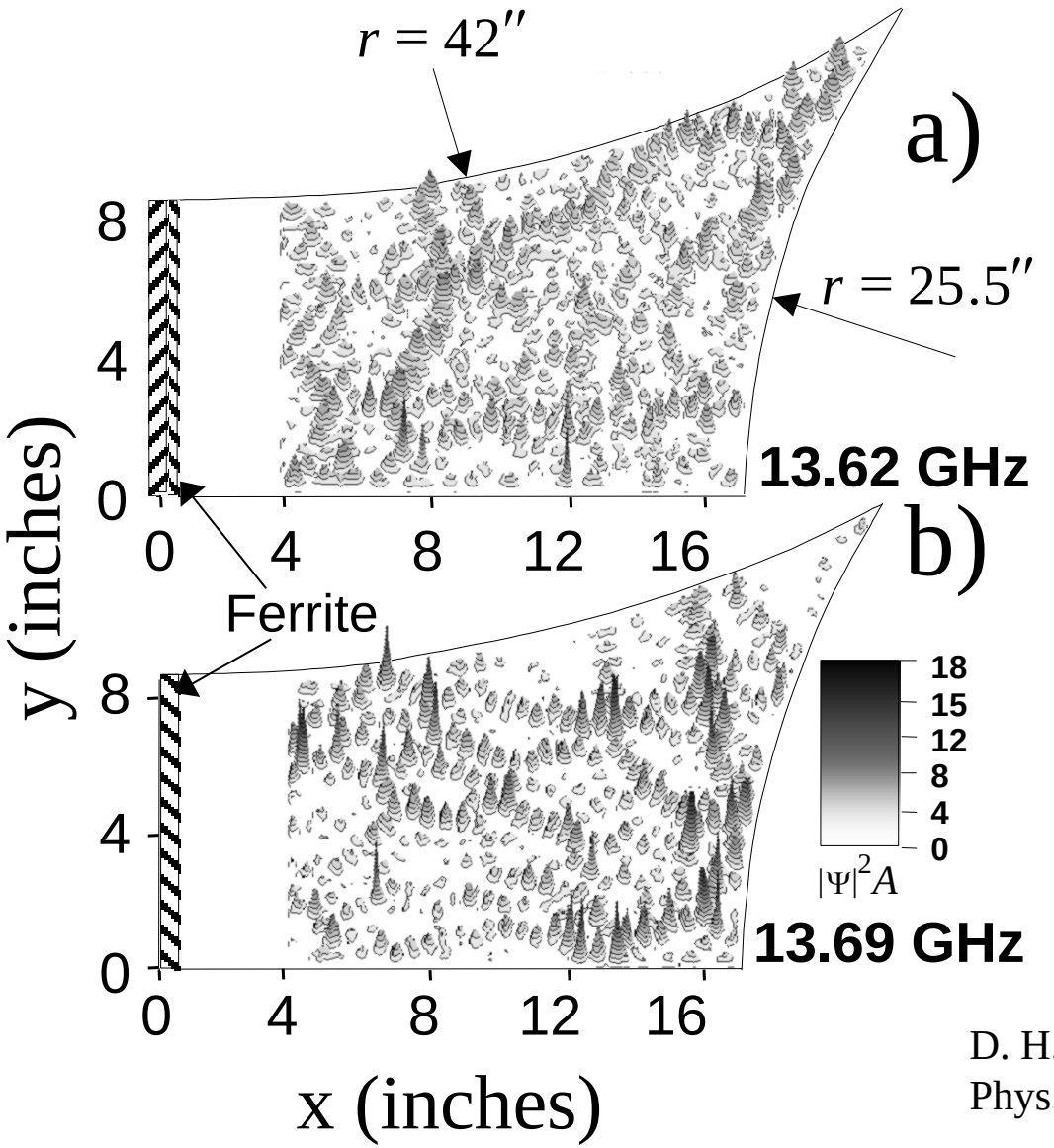
GOE

GOE – GUE Crossover

GUE



Wave Chaotic Eigenfunctions with and without Time Reversal Symmetry



TRS Broken (GUE)

TRS (GOE)

D. H. Wu and S. M. Anlage, Phys. Rev. Lett. 81, 2890 (1998).